VARRO'S MATHEMATICAL MODELS OF INFLECTION

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In Book Ten of his *De Lingua Latina* Marcus Terentius Varro formulated four mathematical models of inflection. My purpose is quite straightforward—to analyze and discuss those passages of the *De Lingua Latina* in which Varro postulated an intimate relationship between mathematics and language.[1]

Traditionally Varro's purpose or intent, at least insofar as Books VIII, IX and X are concerned, used to be defined in terms of an analogy/anomaly quarrel in antiquity. We are now less certain about the reality of such a quarrel and whether it matters very much anyway.[2] At best analogy and anomaly are really little more than cover symbols for a more important issue, because the question is whether language can be described scientifically, that is, by statable principles. The primary issue is therefore theoretical and heuristic in a very real sense, and Varro is well aware of this, as we shall see. In any case the analogy/anomaly quarrel, or more correctly Varro's discussion of the claims of anomaly (Book VIII) and of analogy (Book IX), provides him with a point of departure at the beginning of Book X, and it is there that Varro himself states his intent.

Varro begins by referring separately to similarity and dissimilarity and to analogy and anomaly and then states: . . . *quarum rerum quod nec fundamenta, ut debuit, posita ab ullo neque ordo ac natura, ut res postulat,*

[1] In Daniel J. Taylor, *Declinatio: A Study of the Linguistic Theory of Marcus Terentius Varro* (Amsterdam 1974 [1975]) 42–49, I examine this relationship from a different perspective.

[2] See especially Detlev Fehling, "Varro und die grammatische Lehre von der Analogie und der Flexion," *Glotta* 35 and 36 (1956 and 1957) 214–70 and 48–100 (respectively), and Jean Collart, "Analogie et Anomalie," *Entretiens sur l'antiquité classique* IX (1962) 117–32; cf. my remarks (above, note 1) ix–x and 51–52.

explicata, ipse eius rei formam exponam. Varro[3] explicitly states that he intends to discuss the *fundamenta* of his subject, and that is precisely what he does; "The first steps of most of the sciences are purely classificatory,"[4] and in Book X Varro does analyze the essential principles of linguistic classification. In very simple terms Varro's purpose is to classify words, since according to the tenets of his theory they are the smallest indivisible elements of language: *Verbum dico orationis vocalis partem quae sit indivisa et minima* (X.77). But since there is no existing system of classification, Varro must establish the fundamental issues, theoretical and practical, involved in classifying words, and to this extent Varro is creating information. To discuss classification in this sense is Varro's intent in Book X, and it is within the context of this discussion that Varro formulates his mathematical models of inflection. It is Varro's search for the linguistic principles underlying paradigmatic schemata which I wish to reconstruct in order to determine the proper analysis of his models, for only within the context of his overall discussion can we understand and appreciate his usage of mathematical models of inflection.

Since *similitudo* is the basis of any and all comparison and classification, Varro first addresses himself to the topic of similarity in general (§§3–5). The primary question regarding *similitudo*—and from the point of view of the methodology of linguistic inquiry the crucial issue—is *cui parti* (§6). In what part is similarity to be sought? What is and what is not distinctive? Varro's examples in the next two chapters are instructive. Consider *suis* and *suis*. They appear identical but in point of fact are not, because one is a verb form meaning "you are sewing" and the other is a noun in the genitive case meaning "of a pig." Similarly, *nemus* and *lepus* seem identical but again are not, because *nemus* is neuter and *lepus* is masculine. Varro's question of *cui parti* strikes right to the center of the entire problem.[5]

Furthermore, according to Varro, his predecessors had failed to

[3] Varro is serious here, *pace* Jean Collart, *Varron grammairien latin* (Paris 1954) 47 and 134, Antonio Traglia, *M. Terenzio Varrone De Lingua Latina Libro X* (Bari 1956) 52–54 and 165 and *M. Terenzio Varrone La Lingua Latina Libro X* (Rome 1967) 34–35 and 101, and Adriana Della Casa, *Il libro X de De lingua latina di Varrone* (Genoa 1969) 52.

[4] William James, *The Principles of Psychology*, 2 vols. (New York 1890) II, 647.

[5] Varro is well aware of this; referring to *cui parti*, he says (§6) *in hoc enim solet esse error* and in the next chapter adds *is locus maxime lubricus est.*

determine accurately the criteria underlying linguistic similarity (§§9–10). Varro's own approach is elegantly simple; he concludes (§11) that there are two and only two factors involved, *materia* and *figura*, and thus a proper system of classification must be predicated on a *similitudo* of words which is *duplex et perfecta* (§12),[6] that is, one which encompasses both *materia* and *figura*. To generalize in this way is to depart significantly from the customary data-oriented approach to the study of language,[7] and Varro's distinction is no mean accomplishment. What Varro means by *materia* and *figura* is at this point, however, not all that obvious, as he himself realizes (§13), but his subsequent discussion provides definitions for these terms. *Figura* is a common technical term in ancient grammar, but Varro's usage is idiosyncratic.[8] Briefly put, in Varro's metalanguage *figura*, which occurs extensively throughout the *De Lingua Latina*, refers always to the shape, the external appearance, the surface form of a word, and since this is a matter of sounds and letters, a word's phonological configuration is its *figura*. Unlike *figura*, *materia* occurs infrequently—only twice—in the *De Lingua Latina*, but its definition is not difficult to ascertain. As Varro's examples above indicate, the set of morphological features which constitute any given word is essential to determining whether it is similar to another word, and thus *materia* refers to a word's grammatical content, as in X.36 where a difference in number is a difference in *materia*. *Figura* is the phonological form, *materia* the grammatical content. The distinction is essential, because *similitudo* exists only when there is a twofold and total conformity of the underlying *materia* and the overt *figura*. The two terms are within his theoretical framework necessary and sufficient to answer the question of *cui parti*.

Varro next, by way of elaborating his prerequisites for similarity and in order to reestablish his point of departure, formulates three empirical *divisiones* (§§14–17) within the corpus of data with which he is working.

[6] Cf. Varro's arguments in X.63–68 where *duplex* (§§63 and 68) and *perfecta* (§68; also §69) reoccur, and Taylor (above, note 1) 92–99.

[7] See R. H. Robins, "Theory-Orientation Versus Data-Orientation: A Recurrent Theme in Linguistics," *Historiographia Linguistica* I (1974) 11–26.

[8] For the customary usages of *figura* in Roman grammar, i.e., as a calque of σχῆμα, cf. the term as used by Diomedes (*GL* I, 301) and Donatus (*GL* IV, 363) with its usage in Charisius (*GL* I, 7) and Priscian (*GL* II, 7–9). Varro stands outside the tradition or paradigm in more ways than one.

At this point in his investigation Varro's corpus is the entire inventory of lexical items in Latin, and these *divisiones* represent an ordered (in the modern sense) series of steps or operations whereby words may be separated initially for the purpose of classification. Therefore, because Varro is trying to establish a system whereby he can classify words which vary in form, the first step or *divisio* is to eliminate those words which never vary in form, i.e., indeclinables. By this first *divisio* Varro formally determines that the investigation must concern itself only with those words which change in form.[9] The second step is to distinguish between *declinatio voluntaria*, what we mean by derivational morphology or word formation, and *declinatio naturalis*, that is, inflectional morphology, and by means of this *divisio* to eliminate from consideration the morphological process which relates words derivationally rather than inflectionally. Words are, to be sure, varied in form to create new words, but Varro is not here concerned with that process. Only *declinatio naturalis* is at issue. The distinction, one may say, is obvious, but Robins points out that it was "not commonly made in antiquity" and considers the dichotomy "one of Varro's most penetrating observations."[10] The third step or *divisio*, also obvious but also previously unstated, is to insist that classification be directed only at words which are the same part of speech; that is, there is an inherent difference between the several *partes orationis* which, unless accounted for, will preclude the linguist from arriving at a correct system of classification. By parts of speech Varro does not mean the set of banal distinctions with which we are all too familiar; rather, he distinguishes his four *partes orationis* on purely formal grounds, as is necessary: (1) words with case, (2) words with tense, (3) words with both, (4) words with neither.[11] Varro concludes by noting the obvious, namely, that

[9] This first *divisio* dichotomizes between what Varro elsewhere (VIII.9) terms the *genus sterile* and the *genus fecundum*, which are so termed because the former *ex se parit nihil* and the latter *declinando multas ex se parit disparilis formas*.

[10] R. H. Robins, *A Short History of Linguistics* (Bloomington and London 1967) 50. As obvious as it is, the distinction is nevertheless often overlooked or not maintained, not only by Varro himself on one occasion (X.28) but also by modern linguists.

[11] Varro's examples for these four *partes orationis* here are respectively: (1) *docilis, facilis*; (2) *docet, facit*; (3) *docens, faciens*; and (4) *docte, facete*. The adverbs here (but not those monomorphemic ones which are subsumed under the *genus sterile*; cf. note 9 above) are grammatical inflections and are therefore subject to analysis in terms of *declinatio naturalis*; cf. Robins (above, note 10) 51 and Vladimir Karakulakov, "'Pergamskaja' i

classification may proceed within any one, but only that one, part of speech. His methodological point is clear.

Varro now proceeds to exemplify the grammatical distinctions which accrue to the several parts of speech he has distinguished, but although there is much of interest in these passages, only a few points need concern us. Varro observes (§§21–24), for example, that nouns, in order to be classified together, must be of the same number, gender, case, and kind (common or proper); that is, the grammatical substance must be identical, and of course they must have the same ending or phonological form. Furthermore, more than just a single member of the paradigm must be taken into consideration (§29), and thus two sets of at least two forms, i.e., four words at the minimum, are involved in even the simplest test of *similitudo*. After touching upon the other parts of speech[12] Varro sums up his discussion with this example (§36): "The same relationship [*ratio*] is in *amor amori* as is in *dolor dolori*." Form and substance are identical respectively. Most importantly for our purpose Varro notes that the relations here are proportional and states that classification demands this type of proportional similarity.

Varro then turns to the term *ratio pro portione* and in what appears to be a translation from a Greek handbook defines (§37) it and some other terms and explains what we mean by the notation $a:b::c:d$, that is, "a" is to "b" as "c" is to "d," or as Kent notes, "As in mathematics, two ratios of equal value make a proportion."[13] But what specifically interests Varro at this stage of his discussion is the systematic relationship, the *logos* or *ratio*, between the two members of each set, and his first example is as interesting as it is informative: *Nam ut in geminis, cum similem dicimus esse Menaechmum Menaechmo, de uno dicimus; cum similitudinem esse in his, de utroque* . . . (§38). Menaechmus, Varro says,

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morfologičeskaja klassifikacija častej reči u Varrona," *Učenye zapiski Dušanbinskogo gosudarstvennogo pedagogičeskogo instituta im. T. G. Ševčenko, Filologičeskaja serija* 33, 18 (1965) 113–34. Let me record here my conviction, not shared by others, that Varro's examples are with but few exceptions uniformly well chosen.

[12] For Varro's analysis, such as it is, of verbs, see H. J. Hartung, "*παρεπόμενα ῥήματος* bei Varro?" *Glotta* 51 (1973) 293–311.

[13] R. G. Kent, *Varro on the Latin Language*, 2 vols. with continuous pagination, Loeb edition, revised (Cambridge, Mass. and London 1951 [reprinted 1958, 1967]) 562. Kent's note is correct, but he anticipates Varro's argument; Varro does not introduce any numbers until §41 where they are but one of four examples.

is similar to his twin brother, also named Menaechmus, but a descriptive statement of this sort refers only to the one twin; if, however, they both have an attribute, characteristic, similarity, or likeness in common, then any statement based on that common feature is valid for both twins. If a *ratio* exists, then whatever is predicated of one set applies equally to the other set; or as Varro puts it (§40), . . . *cum dixero quid de utroque, et erit commune.* The principle is of wide validity as Varro demonstrates[14] in the next two chapters when he mentions numbers, coins, the family (son and father, daughter and mother), time (noon and day, midnight and night), similes in poetry, geometry, and grammar. Grammatically it is clear where he is heading: a proper system of linguistic classification entails—and I use the verb in its logical sense—that anything which can be said of one member or word can be said of any and all others within that class. It is at this point that Varro devotes the remainder of his discussion (§§43–50) to mathematics and language and draws parallels between the *ratio* of linguistic forms and the *ratio* of numbers expressed proportionally, and it is here that he formulates his mathematical models of inflection.

Of the four models which Varro formulates,[15] the simplest is that which represents the *deiunctum* "disjoined" type of *analogia.* Varro describes it as follows: *ut unum ad duo, sic decem ad viginti*; that is, one is to two as ten is to twenty, or schematically 1:2::10:20. The numbers are all different, but the ratio is identical since the second member of each set is the double of the first member. Together they therefore constitute an *analogia* or mathematical proportion. The disjoined mathematical proportion is linguistically significant, because it parallels or represents the linguistic *ratio* and *analogia* which exist in nouns: *ut rex regi lex legi.*[16] *Rex* is to *regi* as *lex* is to *legi.* The words are all

[14] Since Varro's readers were not likely to react favorably to a discussion based on mathematics, these non-linguistic and non-mathematical parallels are by no means unimportant for his presentation.

[15] It is for the reader's convenience as well as my own that in what follows I depart slightly from the order in which the models are presented in the text by deferring until last my discussion of the model which actually comes first; on the other hand a possible dislocation of the text—and there are several in the *De Lingua Latina*—is at least thinkable, and whoever chooses to infer that I think that way is invited to do so. In any case I begin with Varro's simplest model.

[16] Karl Müller, *M. Terenti Varronis De Lingua Latina Librorum Quae Supersunt* (Leipzig 1833) 252, added *lex legi* to the text here, and his emendation has been accepted.

different, but the grammatical *ratio* is identical since the second member of each set is related both phonologically (cf. *figura*) and morphologically (cf. *materia*) to the first member in the same way. Together they therefore constitute an *analogia* or linguistic proportion, and this is what I term a mathematical model of inflection.

Another type of analogy is *coniunctum* "conjoined:" *ut unum ad duo, sic duo ad quattuor*; one is to two as two is to four, or schematically 1:2::2:4. It is called conjoined, because the number 2 is repeated since it is to be compared with both 1 and 4 and participates in each ratio. But again the ratio is identical in each set and is again the concept of the double, again we have a valid mathematical proportion, and again that proportion, though conjoined, is still fourfold. Varro exemplifies what he means by *quadruplex* "fourfold" by referring to music and medicine,[17] and the point of his parallels is that even though only three items are at issue the relationship between them is proportional and fourfold since the middle item participates in both ratios. Consequently we may relate the three numbers 1, 2, and 4 in a four-fold mathematical *analogia* by conjoining the number 2 with both 1 and 4. Therefore, 1:2::2:4.

The linguistic *analogia* is found, according to Varro, in the verb. The verb forms *lego*, *legebam*, and *legam* are proportionally related in the following (schematic) manner: *legebam*:*lego*::*lego*:*legam*; as Varro says, *legebam* has the same *ratio* to *lego* as *lego* has to *legam*. In other words, the past is to the present as the present is to the future. The proportion is fourfold, because the present *lego* is conjoined with both the past *legebam* and the future *legam*. Varro was convinced that the vast majority of investigators had failed to determine the nature of verbal relationships correctly, because they did not conjoin the present with the past and the future: *In hoc fere omnes homines peccant, quod perperam in tribus temporibus haec verba dicunt, cum pro portione volunt pronuntiare* (§47). They did not realize the inherently fourfold nature

[17] Two points need to be made about these parallels. First, since Varro relates musical harmonies and numbers, one suspects that there is at least some Pythagorean inspiration for the parallel; Collart (above, note 3) 37–38 collects instances of Pythagorean influence on Varro, and we should perhaps add this passage to his list. Second, in the medical parallel Varro says that the doctor predicts (*praesagit*) the unknown (i.e., the future course of an illness), and this is precisely what he later (§51; cf. §59) concludes the grammarian can do in the case of words if, that is, there is in fact similarity.

of the linguistic relationships in the verb which are parallel to the conjoined mathematical proportion. There is, however, more at issue in the linguistic *analogia* than a simple failure to arrange the data, i.e., verbs, in the correct proportional schema, because the major reason for that failure, according to Varro, has to do with the fundamental character of the Latin verb. The Latin verb system contains a distinction in aspect between the *infectum* and the *perfectum*, to use Varro's terms, and this distinction is of prime grammatical importance.[18] But Varro's predecessors had not observed this distinction and had therefore not used it in analyzing the verb. Since some forms are *infecta* and others *perfecta*, they ought to be conjoined only with other verb forms of their own class; the imperfective *lego*, says Varro, can be correctly related to the imperfective *legebam*, but not to the perfective *legi*. This is . . . *quod et infecti inter se similia sunt et perfecta inter se, ut tundebam tundo tundam et tutuderam tutudi tutudero; sic amabar amor amabor, et amatus eram amatus sum amatus ero*[19] (§48). Schematically these four verbal *analogiae* are as follows:

tundebam : *tundo* : : *tundo* : *tundam*
tutuderam : *tutudi* : : *tutudi* : *tutudero*
amabar : *amor* : : *amor* : *amabor*
amatus eram : *amatus sum* : : *amatus sum* : *amatus ero*

Consequently the grammarian may formulate a linguistic *analogia* or proportion only within one or the other aspect—progressive or imperfective verb forms are not to be conjoined with perfective ones. The correct verbal proportion is conjoined and must take aspect into account.

A third mathematical proportion is rather interesting if for no other reason than that it does not provide a correct model for the grammatical data simply because it is not possible to do so. Varro states that analogy seems sometimes to have fewer parts (as in the conjoined verbal

[18] Robins (above, note 10) 51–52 and *Ancient & Mediaeval Grammatical Theory in Europe* (London 1951) 56–57 understands Varro's analysis of aspect but criticizes it; K. C. Masterman, "On Grammatical Terminology and Aspect in Particular," *G&R* 9 (1962) 75–77, offers a more positive assessment.

[19] Although we take the future perfective indicative forms for granted, by far the greater part of the history of Latin scholarship did not but instead considered them as subjunctives; for an historical account of this issue see Aldo Scaglione, *Ars Grammatica: A bibliographical survey, two essays on the grammar of the Latin and Italian subjunctive, and a note on the ablative absolute* (The Hague 1970) 44–111.

proportion); similarly at times it seems to have more as when it is thus: *ut ad tria unum et duo, sic ad sex duo et quattuor*, that is, 1 and 2:3::2 and 4:6. Varro considers this proportion fourfold, because sets of two (1 and 2, 2 and 4) are compared with sets of one (3, and 6). In speech, he says, this can sometimes be found, as when nominative *Diomedes* is compared with genitives *Diomedi* and *Diomedis*, and similarly from nominative *Hercules* there are genitives *Herculi* and *Herculis*. The linguistic *analogia* is therefore *Diomedi* and *Diomedis*:*Diomedes*::*Herculi* and *Herculis*:*Hercules*. Varro's grammatical referents are what we call heteroclites, substantives which show different stems from the same nominative. It is a type of word whose functional yield or role in the overall linguistic system is minimal at best, but as Varro notes elsewhere (VIII.26), they do occur and are in use. In any case, to return to the putative parallel between language and mathematics, it is obvious that here the parallel simply does not exist. In the mathematical proportion the numeral 2 is repeated, but in the linguistic proportion, no item is repeated. This, however, is not really crucial, for even if we revised the mathematical model so as to eliminate the repeated cipher and yet retain the proper ratios, we still could not match the numbers and the words. The linguistic *analogia* demands two words which bear an equivalent *ratio* to a third word but which are themselves not equivalent, and there can be no parallel for this in simple numerical relationships. Either of the two existing inflectional patterns could by itself fit the disjoined *analogia* of nouns, but when they are taken together, no correct mathematical model is possible. There is no need to extend discussion of this model by engaging in arithmetical gymnastics, and we may take leave of it by merely reiterating that the mathematical and linguistic proportions do not match as the others have.

The one remaining mathematical model of inflection contains more numbers than the others but is really not at all complicated. Varro notes that analogy sometimes has two relationships intertwined in such a way that one is vertical and the other horizontal, and he sketches in prose the following schematic set of arithmetical proportions: 1:2:4 as 10:20:40 as 100:200:400. We may diagram this as:

1	2	4
10	20	40
100	200	400

There is a horizontal relationship, the double, and also a vertical ratio, the tenfold, and throughout the proportion or model both ratios are constant. The linguistic model occurs in the following chapter where Varro supplies adjectival forms[20] in place of the numerals; that is:

hic albus	*huic albo*	*huius albi*
haec alba	*huic albae*	*huius albae*
hoc album	*huic albo*	*huius albi*[21]

Varro, to point out the obvious, arranged the members of his paradigmatic sets, such as they were, differently than we do. The parallel with mathematics is apt,[22] for there are two sets of *rationes* in the adjectives just as in the numbers; the horizontal items represent the systematic relationship between the cases, while the gender relationships are expressed vertically. Both grammatical *rationes* (case and gender) are constant throughout the model.

Those are Varro's four mathematical models of inflection. They are a perfectly natural and logical culmination to his discussion of inflectional morphology and his search for a system of classification. Descriptively the conclusion is clear—words, in order to be classified together, must exhibit, in terms of their phonological form and morphological content, the same type of exact relationships as numbers in mathematical proportions. Varro observed the systematic relationships inherent in inflectional morphology and described and explained

[20] Earlier (§22) Varro had introduced the adjectival paradigm into his discussion and had compared its arrangement to that of a game: *ut in tabula solet in qua latrunculis ludunt*; but unfortunately the text breaks off in the middle of the next chapter (*hic desunt tria folia in exemplari*, says the second hand in the margin), and consequently, although the context allows us to determine the general topics Varro covered in the lacuna (see Kent [above, note 13] 550–51), we do not know how he used the adjectival arrangement in his discussion.

[21] It is not especially clear whether we should retain the forms of the *articulus* with the adjectival forms here, as Traglia and Della Casa (above, note 3) do in their translations, or whether with Kent (above, note 13) we should render them grammatically, i.e., *hic* = nom. sg. masc.; Varro does, however, include them here whereas he had not done so in §22.

[22] The parallel may be apt, but it is also forced since even the presence of the *articulus* cannot everywhere distinguish between those forms whose *materia*, but not their *figura*, differs. Therefore the numbers, but not the forms, are all different, and the linguistic *analogia* does not match the mathematical model exactly. Both *analogiae* do, however, manifest the two intertwined *rationes*, and since these were Varro's point of departure for this model, perhaps we should not overanalyze it.

them by drawing parallels between the *ratio* of numbers expressed proportionally and the *ratio* of linguistic forms. The mathematical proportion exemplified fully the systematic regularities he had discerned in inflection. By basing linguistic classification on complete paradigmatic identity, Varro was arguing for morphophonemic patterns, that is, embryonic declensions and conjugations, which were as precise and as exact as numerical proportions. Varro's rigid constraints and his insistence on absolute similarity are now discarded for obvious reasons, but they may be recognized as the initial stages of any scientific, classificatory investigation. Moreover, to quote an historian of science: "Classification underlies definition and generalization, and, since these constitute the frame of knowledge, science necessarily builds upon them."[23] As I stated earlier, Varro created information. He created information when he argued in an orderly and methodologically explicit manner that *analogia* is a fact of language, just as it is of mathematics.[24] I conclude by suggesting that Varro's mathematical models of inflection bear witness to his originality in linguistic inquiry.

[23] William Arthur Heidel, *The Heroic Age of Science* (Baltimore 1933) 118.

[24] Rudolph Pfeiffer, *History of Classical Scholarship* (Oxford 1968) 203, categorically declares that the grammatical term ἀναλογία "is hardly derived from the mathematical and philosophical term ἀναλογία," but Raoul Mortley, "Ἀναλογία chez Clément d'Alexandria," *REG* 84 (1971) 81, is much more likely to be correct when he states: "Le sens mathématique est fondamental: il constitue le facteur commun du terme ἀναλογία pendant toute son histoire en grec." The latter point would be particularly appropriate in the case of Varro if, as Jacques Heurgon, "L'Effort de style de Varron dans les *Res Rusticae*," *RPh* 24 (1950) 70, suggests, he was in fact subject to the "influence tyrannique du mot." At any rate Varro formally postulated an intimate relationship between the *analogia* of mathematics and that of language.